1. Determine all solutions of the system

$$\begin{cases} x - y - z = 0 \\ 2x + 5y - 3z = 13 \\ -3x + 4y = -1. \end{cases}$$

2. Determine all a such that the system

$$\begin{cases} x + ay = a \\ ax + 4y = 2a \end{cases}$$

is inconsistent.

3. Consider the matrices $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

   We remark that $B^{-1} = B$.

   Determine $A^{-1}, A^{-1}B^{-1}$, and $(AB)^{-1}$.

4. Determine all b such that the matrix

$$B = \begin{bmatrix} b & 0 & 0 & 2 \\ 0 & \sqrt{b} & 0 & 0 \\ 1 & 2 & 0 & 7 \\ 3 & 4 & 5 & 6 \end{bmatrix} \text{ is singular.}$$

5. Determine $1 + 2 + 3 + \ldots + 999 + 1000$.

6. Determine a (particular) solution of the recurrence relation

   $x_t = 3x_{t-1} + 2x_{t-2} + 4t - 3, \quad t \geq 2.$

7. It is given that -1 and 2 are equilibria of the recurrence relation $x_t = cx_{t-1}^2 + d$. Determine c and d. Which equilibria are stable, and which are not?

8. Consider the Markov chain with transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} \\ 0 & \frac{1}{2} & \frac{3}{4} \end{bmatrix}. \text{ Determine the steady states of this Markov chain.}$$

9. Solve the following LP-problem graphically:

$$\begin{array}{lrcrcr}
\min. & x_1 & - & x_2 & & \\
\\
\text{s.t.} & x_1 & + & x_2 & \geq & 4 \\
 & x_1 & + & 2x_2 & \leq & 18 \\
 & 2x_1 & - & x_2 & \geq & -4 \\
 & 3x_1 & - & x_2 & \leq & 12.
\end{array}$$

10. Solve the following LP-problem, and determine the shadow prices:

$$\begin{array}{llrcrcrcr}
\max. & z = & x_1 & + & 2x_2 & + & 3x_3 & & \\
\\
\text{s.t.} & & x_1 & + & x_2 & + & 2x_3 & \leq & 9 \\
 & & x_1 & + & x_2 & + & x_3 & \leq & 5 \\
 & & 2x_1 & & & & x_3 & \leq & 4 \\
\\
 & & x_1 \geq 0, & & x_2 \geq 0, & & x_3 \geq 0. & &
\end{array}$$

11. Consider the cooperative game (N, v), with $N = \{1, 2, 3\}$ given by:

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$v(S)$	5	4	7	e	14	12	18

Determine the largest e for which the core of this game is not empty, and determine a core vector for that e.

12. Consider the linear production game for a production situation with 2 ingredients and 3 products, given by the technology matrix

$$A = \begin{bmatrix} 2 & 6 & 3 \\ 2 & 3 & 6 \end{bmatrix} \text{ and price vector } c = \begin{bmatrix} 60 \\ 150 \\ 90 \end{bmatrix}.$$

There are two players with resource vectors $b_1 = \begin{bmatrix} 30 \\ 10 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 20 \\ 30 \end{bmatrix}$.

Determine an Owen vector of this game.

13. Determine the value of the matrix game $A = \begin{bmatrix} 5 & 1 \\ 2 & 4 \end{bmatrix}$.

14. A firm wants to manufacture the products 1, 2, 3, and 4. It has 6 machines A, B, C, D, E, F available to run this process. Moreover, each machine can be used to manufacture at most one of the products 1, 2, 3, 4.

    The production time to manufacture a product on a certain machine is displayed in the following table:

	A	B	C	D	E	F
1	2	2	4	7	8	4
2	3	2	5	5	5	5
3	4	2	6	5	4	4
4	2	3	4	4	4	3

    Determine an assignment of the products 1, 2, 3, 4 to the machines such that the total production time is minimized.

15. Two players play a variation of Tic Tac Toe on a piece of 8×8 squared paper. The players alternate moves, where each move the first player (who starts) puts an $\times$ in one of the empty squares, and the second player puts an O in one of the empty squares. The player who first fills a 2×2 square with the same symbol wins.

$\times$	$\times$
$\times$	$\times$

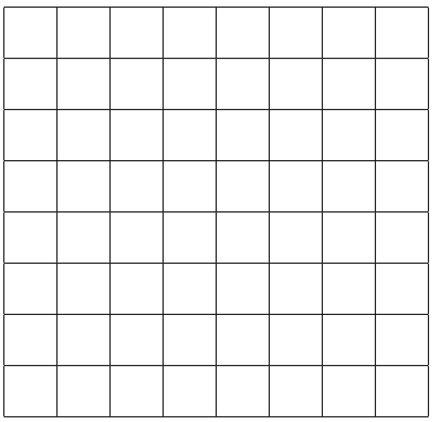
    : player one wins;

O	O
O	O

    : player two wins

a) Fill the 8×8 squared paper with 32 bricks of the form

   (each filling two squares) such that each of the 2×2 squares on the paper contains at least one whole brick.



b) Give a strategy for player two that prevents player one from winning.