This examination paper
has 5 numbered pages

ERASMUS UNIVERSITY ROTTERDAM
Entrance examination Mathematics
for Business Administration

PRACTICE EXAM

- This entrance examination consists of two parts. Each question of Part I is worth at most three points, and each Part II question at most twelve points. In order to pass the examination a score of at least eighteen points is required.
- The use of a so called "graphing calculator" or "programmable calculator" is not permitted. "Simple" scientific calculators are allowed.
- All necessary steps, formulas, substitutions, diagrams or graphs leading to your final answer must be written down. Furthermore, questions containing the words "solve", "compute" or "calculate" require an exact answer; a decimal approximation is not allowed.

Part I. Please complete <u>exactly eight</u> of the following fifteen questions.

Question 1

(a)   Solve the equation $3x^2 - 10x + 3 = 0$.

(b)   For which value(s) of p does the equation $px^2 - 10x + p = 0$ have exactly one solution?

Question 2

(a)   Sketch, in the same figure, the graphs of the functions

$$f : \ x \to \frac{x}{2} \ \text{and} \ g : \ x \to \ \frac{4}{\sqrt{x}}.$$

(b)   Compute the point of intersection of the graphs of f and g.

(c)   Determine the solution of the inequality: $f(x) \leq g(x)$.

Question 3

Determine the derivative, with respect to the variable x, of each function below:

(a) $f(x) = \frac{e^x}{1+e^x}$

(b) $f(x) = \ln(1 + x^2)$

Question 4

Solve each of the systems of equations below.

(a) $\begin{cases} 3x - 5y &= 19 \\ -5x + 3y &= -5 \end{cases}$

(b) $\begin{cases} 2x - y &= 1 \\ x + \frac{1}{4}y &= -1 \end{cases}$

Question 5

(a) Sketch the graph of $f : \ x \to e^{2-x}$.

(b) Determine the slope of the line tangent to the graph of f at the point $(2, 1)$.

Question 6

(a) In the (x, y)-plane draw the region given by

$$y \le \ln x \ \text{ and } y \ge -\ln x \ \text{ and } \ x \le e.$$

(b) This region has three vertices A, B, and C. Give the (x, y)-coordinates of these vertices.

Question 7

Solve the following equations:

(a) $x^{2/3} = 25$

(b) $\left(\frac{1}{4}\right)^x = 64$

(c) $x^{1/5} = \frac{1}{4}$

Question 8

Given is the function $f : \ t \to \ 4\sin\frac{\pi}{2}(t - 3)$.

(a) Calculate the zeros of f.

(b) Sketch the graph of f.

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Question 9

Given are the matrices $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} p & 5 \\ q & 11 \end{bmatrix}$.

(a)   Compute p and q subject to $AB = BA$.

(b)   Compute the determinant of matrix A.

Question 10

Given is the function $f : x \rightarrow |2x-1| - |x-2|$. Sketch the graph of this function.

Question 11

Capital K is deposited at an annual interest rate of 4%. Interest is compounded once a year. In fifteen years time the capital has grown to an amount of 108056.62 euro. Compute the principal capital K.

Question 12

Of a group of one thousand students who did the examination for Mathematics as well as the examination for Statistics, it is known that 600 of them passed the Mathematics exam and 650 passed the Statistics exam. Moreover, 400 students passed both exams.

(a)   How many students failed both exams?

(b)   How many students passed one and failed the other exam?

Question 13

A bacterial population grows according to the formula $N_t = 500e^{0.069t}$, where t is measured in days.

(a)   Calculate N_0.

(b)   Determine the value of t for which $N_t = 2N_0$.

Question 14

(a)   Calculate the possibility of throwing six 6's in six throws with an unbiased die.

(b)   Calculate the possibility of throwing a 6 exactly once in six throws with an unbiased die.

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Question 15

(a) Calculate $\displaystyle\int_0^1 \frac{2x}{1+x^2}\,\mathrm{d}x$

(b) Calculate $\displaystyle\int \frac{x^2 - 2x + 1}{1+x^2}\,\mathrm{d}x$

Part II. Please complete only one of the following two questions.

Question A

Given is the function $f : x \to \frac{1-e^x}{1+e^x}$.

(a) Determine the zero(s) of f.

(b) Solve the inequality: $\frac{1-e^x}{1+e^x} > 0$.

(c) Show that $f(-x) = -f(x)$ for all $x \in \mathbb{R}$.

(d) Compute $\displaystyle\lim_{x \to \infty} f(x)$.

(e) Compute $\displaystyle\lim_{x \to -\infty} f(x)$.

(f) Determine the derivative $f'(x)$.

(g) Solve the inequality: $f'(x) < 0$.

(h) Sketch the graph of f.

(i) Calculate the point of inflection of f.

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Question B

It is claimed by traffic experts that the number of passenger cars (A) that will make daily use of a toll tunnel to be built can be calculated by means of the formula

$$A = 400T^2 - 9150T + 46\,800.$$

Here T stands for the toll fee per car in euros. Toll fees in excess of 7.00 euros are not being considered. In view of a smooth traffic flow through the tunnel, the actual toll fee collection will be carried out electronically. As an added advantage it is then possible to charge unusual fees like 2.67 euros without practical difficulties.

(a) Calculate the total daily toll proceeds for passenger cars at a tariff of 2.00 euros per car.

(b) Calculate the toll fee that should be charged in order to make the daily proceeds maximal.

(c) Calculate the percentage decrease (to two decimal places) in number of passenger cars when the toll fee of 2.40 euros is increased by 5%.

At a certain toll fee, a toll tariff increase of 6% will cause a decrease of 2.8% in the number of passenger cars passing through the tunnel each day.

(d) Calculate the percentage increase (to two decimal places) of the daily toll proceeds for passenger cars caused by this tariff increase.