

This exam consists of  
3 numbered pages

**ERASMUS UNIVERSITY ROTTERDAM**  
**Entrance examination Mathematics level 2**  
**for International Bachelor Economics & Business Economics (IBEB)**  
**Wednesday March 9, 2016**

Please note:

- Available time: 2.5 hours (150 minutes).
- In all your answers, give a complete solution where you show all the required steps, formulas, and substitutions that lead to your answer. A good or wrong answer is only a small part of the solution. The quality and completeness of your detailed solutions determine the points you will get. You should end an exercise with a conclusion or an answer.
- The use of a graphing calculator or of a so called programmable calculator is not permitted. The use of a simple scientific calculator is allowed.

### Question 1

Solve each of the systems of equations below.

$$(a) \quad \begin{cases} 3x + 11y = 35 \\ 2x + 5y = 14 \end{cases} \quad (b) \quad \begin{cases} 5y + 7 = 2x \\ 11y + 21 = 3x \end{cases}$$

### Question 2

Determine the derivative of each of the following functions:

(a)  $f(x) = (2 - x^2) \ln(3x)$

(b)  $f(x) = \frac{e^{3x}}{2 - x^2}$

### Question 3

Given the function  $f(x) = e^{-2x^2+x}$

- (a) Determine  $f'(x)$ .
- (b) Determine the formula of the line tangent to the graph of  $f$  at  $x = \frac{1}{2}$ .
- (c) Determine the extreme values of  $f(x)$  or show that  $f$  has no extreme values. Determine for each extreme value whether it is a (local) maximum or a (local) minimum.
- (d) Show that  $f''(x) = (16x^2 - 8x - 3)e^{-2x^2+x}$ .
- (e) Determine the points of inflection of  $f$  or show that  $f$  has no inflection points.
- (f) Sketch the graph of  $f$ , based on the answers to the previous questions.

### Question 4

Solve the following equations:

(a)  $\ln(x^3) = \ln(x^6 - 7x^3)$

(b)  $\sqrt{2(x^2 + x)} - x = 4$

(c)  $(5^x)^3 = \frac{1}{5^{5-x}}$

### Question 5

Consider the functions  $f(x) = 4 - |x|$  and  $g(x) = 8 - x^2$

Note:  $|x|$  denotes the absolute value of  $x$ .

(a) Sketch the graphs of  $f$  and  $g$  in one figure.

(b) Solve  $4 - |x| \leq 8 - x^2$

### Question 6

(a) Determine the formula of the straight line through the point  $(10, 3)$ , and perpendicular to the line  $5y - 2x = 10$ .

(b) Determine the formula of the straight line through the point  $(10, 3)$ , and parallel to the line  $5y - x = 10$ .

(c) Determine the formula of the straight line through the points  $(3\frac{3}{4}, -1\frac{1}{4})$  and  $(-5\frac{1}{4}, 1\frac{3}{4})$ .

### Question 7

(a) Sketch in the same figure the graphs of the functions

$$f(x) = \frac{x + \frac{1}{2}}{2} \quad \text{and} \quad g(x) = \frac{2}{x + \frac{1}{2}}$$

(b) Calculate all points of intersection of the graphs of  $f$  and  $g$ .

(c) Determine all the values of  $x$  for which the inequality  $f(x) \geq g(x)$  holds.

### Question 8

(a) Solve the equation  $x^2 + 10x + 10 = 1$  and solve  $x^2 - 8x - 8 = 1$

(b) Determine all values of  $p$  for which the equation  $x^2 + px + p = 1$  has exactly 2 solutions.

### Question 9

Consider the function  $f(x) = ax^4 - 16x^3 + b$ . Assume that  $(x, y) = (2, 6)$  is an inflection point of this function. Show that  $f$  has another inflection point and compute the  $x$ - and  $y$ -coordinate of this other point of inflection.